# Introduction to the abacus

You may recall the abacus from primary school as an aide for counting, using columns of beads. But the abacus is not just for primary school, you can use it for serious arithmetic calculations without the need for paper or electronic calculators, or you can use it as a numerical notepad.

You could use it to note down telephone numbers, the date, or keep scores in games. Some people might like to use it just to keep the mind more active.

## Physical Description

The basic principle of an abacus is a series of columns with beads or counters that can be slid along. We'll look at a tactile abacus, sometimes called the Cranmer abacus, which is based on the Chinese abacus.

Place the abacus in front of you with the long side towards you. Notice that it consists of a series of rods with beads that move up and down. There are four beads in one section and one bead in another, with a horizontal bar separating the two sections. The four beads should be in the front section towards you, and the single bead at the back, away from you, so the bar is towards the back.

A tactile abacus has a foam backing behind the rods to prevent the beads from slipping. Notice that the bar has tactile marks every three columns, these have their usual meaning to break up large numbers into groups (millions, thousands, and so on).

## Setting Numbers

The four beads below the bar each have a value of 1. The single bead above the bar has a value of 5. Beads gain their value when moved towards the bar.

Start by clearing the abacus (if it is not clear already): move all the lower beads to the front of the frame and all the upper beads to the back of the frame, all beads away from the bar. This is the value 0.

Now, on the leftmost column, move a single bead from the group of four, up towards the bar. This is value 1.

In the next column, move two beads up towards the bar, the value 2.

Similarly set three and four beads in the next two columns.

For the value 5, in the fifth column, move the top bead down towards the bar, keeping all the lower beads away from the bar.

For the value six, move the top bead down and one of the bottom beads up towards the bar.

For seven, it's the top bead down and two of the bottom beads up towards the bar.

And similarly, set eight and nine with the top bead down and three and four beads from the bottom towards the bar. So the value of nine is all the beads towards the bar.

Just for practice, re-read the large number you have set on the first nine columns: 123,456,789. ... and there are plenty of columns still spare.

Practise some more, setting some other numbers, for now use any columns you like. Try setting:

17, 83, 256, 308, 573, 812, 954, 1001, 2020.

Some people simply use the abacus as a numerical notepad: you could use it to quickly take down a phone number, or just keep track of the date. Try, for example, setting your own phone number now.

## Basic Addition

The first arithmetic function is addition. You probably learnt it at primary school, sometimes having to write it down in columns. You can do the same on an abacus, but the advantage is, no paper required.

Let's start with the sum: 21 + 68. First, set the number 21 on the two rightmost columns. Then, simply set the second number, 68, on top of what's already there. Immediately, the answer 89 appears.

Let's try another sum: 315 + 163. First, set 315 on the three rightmost columns, then set 163 over the top of the three rightmost columns. Immediately, the answer 478 should appear.

Tip: if you need to, you can optionally set the second number on the leftmost columns, just as a reminder.

## Overflows and Carries

Just as in paper arithmetic, you cannot always set the second number directly on top of the first without causing overflow. In this case, there are so-called "secrets" or "rules" of addition. It's basically a subtraction instead, with adding 1 to the column on the left.

For example, to add 1 to 9, subtract the 9 and add one to the column on the left, giving 10. a shorter way of saying this is:

To add 1 to 9: clear 9, set one left.

Similarly, if you want to add 9 to any number other than 0, you subtract 1 and add 1 to the column on the left:

To add 9 to >0: clear 1, set 1 left.

So 9 is the complement of 1.

Similarly, to add 2 to 8: clear 8, set 1 left;

to add 8 to >1, clear 2, set 1 left.

So 8 is the complement of 2.

And so on:3 goes with 7, 4 goes with 6, and 5 is on its own.

A couple of examples:

Let's try the sum 43 + 78. Start by setting 43 on the two rightmost columns as usual. If you need to, set 78 on the very left as a reminder.

Now, add 8 to the existing 3. This would cause overflow, so the rule for 8 is instead to clear 2 and set 1 left. This now gives 51.

Now add the 7 to the 5. Again, this causes overflow. The rule for 7 is to clear 3 and set one left, giving our final answer of 121.

Just for practice, try doing the same sum the other way around: 78 + 43.

Here's another sum: 456 + 789. Start by setting 456 on the three rightmost columns.

Now, consider adding 9 to 6. It causes overflow. 9 goes with 1, so clear 1 and set one left.

Now add 8 to the middle column: again this causes overflow. 8 goes with 2, so clear 2 and set 1 left.

Finally, add 7 to the left column. Again this causes overflow: 7 goes with 3, so clear 3 and set one left (bringing a fourth column into use).

Hopefully we arrive at the final answer of 1,245.

Again, for practice, try the same sum the other way around: 789 + 456. You should get the same answer.

## Subtraction

Subtraction is very similar to addition, but in reverse.

Try 84 − 31. Start by setting 84 on the two rightmost columns. Then simply move the second number, 31, away from the bar. The answer of 53 immediately appears.

Similar to addition, underflows and borrows occur in subtraction sums, and like addition, similar rules apply.

For example: 72 − 29. 9 can't be directly taken from 2, so instead, add 1 and clear 1 left. Then finish the sum removing 2 from the 6 in the second column, to give the answer of 43.

## Short Multiplication

Multiplication is basically a rapid form of multiple addition, plus the use of ordinary times tables.

Set the first number two columns to the left of where the answer will appear.

Let's try the sum 296 times 3.

Start by setting 296 on the fifth, fourth and third columns at the right, leaving two spare at the right.

The first calculation is 6 times 3: 18. So, set 18 immediately to the right of the 6, then clear the 6.

Move to the next digit: 9 times 3: 27. So set 27 immediately to the right of the 9 and clear the 9.

Finally, 2 times 3: 6. So set 06 immediately to the right of the 2 and clear the 2. Notice the zero, the answer always takes two columns.

The answer appears as 888.

We could continue with long multiplication: the idea is the same but you leave further spare columns to the right for the additional multiplications of each of the digits.

Division can also be done, with the answer appearing two columns to the left.